**MAT 2002: Experiment no. 3.**

**Shape control of the inextensible cable**

A chain or cable with no flexural rigidity, that is supported at its ends, and hanging under the action of a uniform gravitational field only, assumes a static equilibrium curve, is called a catenary.

We consider a cable, which is inextensible, hanging between the two fixed ends A and B, as shown in the Fig. 1. The vertical between the highest and the lowest points of the cable is called *Sag*, while the horizontal distance between two supports A and B is called *Span*.

We assume that the cable is of length *S* and of constant self-weight per unit length , where  is the mass per unit length of the cable, and  is the gravitational acceleration.

The span and sag are denoted by *L* and *h* respectively.



Figure 1. The cable is supported at its ends, and is under the action of gravity.

The main cables carry the road deck and the vehicles on it via a finite number of vertical

cables, so called hangers.

The self-weight of the bridge deck and main cables constitute a large portion of the total load on the main cables. In order to determine the static equilibrium shape of the main cables at a certain temperature, and for the situation where there are no vehicles on the bridge, we assume that the main cables are loaded by gravity and a downwardly directed point force at the position of each hanger. Each point force represents the weight of the portion of the bridge deck that the pertinent hanger is assumed to carry, and, in some cases, also the self-weight of the hanger. It is necessary to be able to predict the horizontal location of each hanger in the strained bridge in order to predict the magnitude and location of each point force that is applied to the main cables. In addition, it is preferable if the horizontal location of each hanger center line, as well as the distances *L* and *h* of the main cables, can be prescribed.

Cables are usually characterised by having high axial tensile rigidity, and virtually no axial compressive rigidity, in the axial direction of the cable. In addition, it is often reasonable to

assume that the flexural rigidity of a cable is negligible. We assume that the tensile force *T* at the cable centreline.

Determination of the static equilibrium shape of a cable can, in some cases, be simplified if

the downwardly directed external loads, which are approximated as point forces in Fig. 2a, can be further approximated by a continuously distributed load, as shown in Fig. 2b.

|  |  |
| --- | --- |
|  |  |
| Figure 2a: Downwardly directed external loads. | Figure 2b: Continuously distributed load. |

We assume that the cable is loaded by the distributed vertical external load *w*(*x*).

To derive the equation of the cable, we consider a cable element as shown in the Fig. 3 below:



Figure 3.Equilibrium of an infinitesimal element of the inextensible cable under gravitational load.

From the figure, it is clear that the elongations in the vertical direction are and .

The elongations in the horizontal direction are and .

For static equilibrium, the sum of the forces must be equal to zero.

That is, the sum of the vertical forces 





 ----(1)

The sum of the horizontal forces 



 ----(2)

Expanding equation (1) we get

 ----(3)

and ----(4)

Assuming that , we have  and .

Plugging in these approximations in equations (3) and (4) we get,

and



On simplification, we get and .

Ignoring the second order terms  in these equations, we get

 ----(5)

and

. ----(6)

Equations (5) and (6) can be written as

 ----(7)

and

 ----(8)

Integrating (8) we get (a constant)

or . ----(9)

Substituting the value of in (7) we get

. ----(10)

We know that , hence equation (10) becomes,

or

 ----(11)

This is the differential equation for a flexible cable.

Parabolic case:

Assuming constant external load, i.e., =K (a constant).

Neglecting the mass of the cable or assuming that the mass of the cable much negligible when compared to that of the bridge.

Placing the origin at the lowest point of the cable as shown in the figure below:



Now the equation of the cable is  ----(12)

under the conditions , .

Integrating (12) twice and applying the conditions (13), we get the solution

.

**Solution by method of variation of parameters:**

Method of variation of parameters enables to find solution of any linear nonhomogeneous differential equation of second order, provided its complimentary function (C.F.) is given / known. Theparticular integral of the non-homogeneous equation is obtained by varying theparameters, i.e. by replacing the arbitrary constants in the C.F. by variablefunctions.

Consider a linear non-homogeneous second order differential equation with constant coefficients

, where  are constants. (13)

Let the complimentary function be of the form , where  are arbitrary constants. This is the solution of the homogeneous equation .

In the method of variation of parameters, the arbitrary constants  are replaced with two unknown functions and .

Let us assume that the particular integral is of the form  (14)

where and .

On putting the values of  and  in (14), we get the particular integral .

Hence the required solution .

**MATLAB CODE**

% Program for solving differential equation of the form

% ay"+by'+cy=f(x), for a, b and c as constants.

clearall

closeall

clc

symsABxm

p=input('Enter the coefficients a,b,c');

f=input('Enter the RHS function f(x)');

a=p(1);b=p(2);c=p(3);

disc=b^2-4\*a\*c;

m=subs(solve('a\*m^2+b\*m+c') );

if(disc>0)

CF= A\*exp(m(1)\*x)+B\*exp(m(2)\*x);

u=exp(m(1)\*x);v=exp(m(2)\*x);

elseif (disc==0)

CF=(A+B\*x)\*exp(m(1)\*x);

u=exp(m(1)\*x);v=x\*exp(m(1)\*x);

else

alfa=real(m(1));

beta=imag(m(1));

CF=exp(alfa\*x)\*(A\*cos(beta\*x)+B\*sin(beta\*x));

u=exp(alfa\*x)\*cos(beta\*x);v=exp(alfa\*x)\*sin(beta\*x);

end

% Method of variation of parameters.

f1=f/a;

jac=u\*diff(v,x)-diff(u,x)\*v; %Jacobian of u and v

P=int(-v\*f1/jac,x);

Q=int(u\*f1/jac,x);

PI=P\*u+Q\*v;

y\_gen=CF+PI;

dy\_gen=diff(y\_gen);

cond=input('Enter the initial conditions x0, y(x0) and Dy(x0)');

eq1=(subs(y\_gen,x,cond(1))-cond(2));

eq2=(subs(dy\_gen,x,cond(1))-cond(3));

[A B]=solve(eq1,eq2);

y=subs(CF+PI)

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**Example 1:** Solve the equation , , .

**MATLAB input**

Enter the coefficients a,b,c[1 -5 6]

Enter the RHS function f(x)sin(3\*x)

Enter the initial conditions x0, y(x0) and Dy(x0)[0 0 1]

**MATLAB output**

y =

(5\*cos(3\*x))/78 - (16\*exp(2\*x))/13 + (7\*exp(3\*x))/6 - sin(3\*x)/78

**Example 2:** Consider the problem of suspension cable  with the conditions, .

**MATLAB code**

%Program for differential equation of a suspension cable

clearall

closeall

clc

symsABxm

W=input('Enter the external load: ');

T=input('Enter the horizontal tension: ');

f=W/T;

a=1;b=0;c=0;

disc=b^2-4\*a\*c;

m=subs(solve('a\*m^2+b\*m+c') );

if(disc>0)

CF= A\*exp(m(1)\*x)+B\*exp(m(2)\*x);

u=exp(m(1)\*x);v=exp(m(2)\*x);

elseif (disc==0)

CF=(A+B\*x)\*exp(m(1)\*x);

u=exp(m(1)\*x);v=x\*exp(m(1)\*x);

else

alfa=real(m(1));

beta=imag(m(1));

CF=exp(alfa\*x)\*(A\*cos(beta\*x)+B\*sin(beta\*x));

u=exp(alfa\*x)\*cos(beta\*x);v=exp(alfa\*x)\*sin(beta\*x);

end

% Method of variation of parameters.

f1=f/a;

jac=u\*diff(v,x)-diff(u,x)\*v; %Jacobian of u and v

P=int(-v\*f1/jac,x);

Q=int(u\*f1/jac,x);

PI=P\*u+Q\*v;

y\_gen=CF+PI;

dy\_gen=diff(y\_gen);

cond=[0 0 0];

eq1=(subs(y\_gen,x,cond(1))-cond(2));

eq2=(subs(dy\_gen,x,cond(1))-cond(3));

A=solve(eq1);

B=solve(eq2);

y=subs(CF+PI)

**----------------------------------------------------------------------------------------------------------------**

**MATLAB input**

Enter the external load: 1

Enter the horizontal tension: 1

**MATLAB output**

y =

x^2/2

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**MATLAB input**

Enter the external load: x

Enter the horizontal tension: 1

**MATLAB output**

y =

x^3/6

**----------------------------------------------------------------------------------------------------------------**

**Exercise problems:**

1. Solve the equation , , .

2. Solve the equation , , .